When Does a Bit Matter? Techniques for Verifying the Correctness of Assembly Languages and Floating-Point Programs

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Framing My Thesis

- I enjoy working with either *no* abstraction or *lots* of abstraction
  - Assembly ☺
  - Java ☹
  - Matlab ☺
- I noticed a couple common abstractions which when they failed were hard to fix
  - Instruction Set Architectures (ISAs)
  - Floating Point (FP)

Some Intuitive Definitions

- *High-level*: using abstractions; not concerned with underlying implementation of a program
- *Low-level*: the opposite

Key Challenge

Abstractions give insight into the nature of a program.

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Dissertation Question

How can we apply high-level reasoning techniques about computer programs to low-level implementations? Specifically,

1. How can we write specifications of instruction set architectures (ISAs) that enable static analysis for program verification?
2. How can we formalize and quantify the error from floating-point arithmetic in high-performance numerical programs?

Quameleon: A Lifter and Intermediate Language for Binary Analysis

Based on previously published work in collaboration with Philip Johnson-Freyd, Jon Aytac, Tristan Duckworth, Michael J. Carson, Geoffrey C. Hulette, and Christopher B. Harrison [6]

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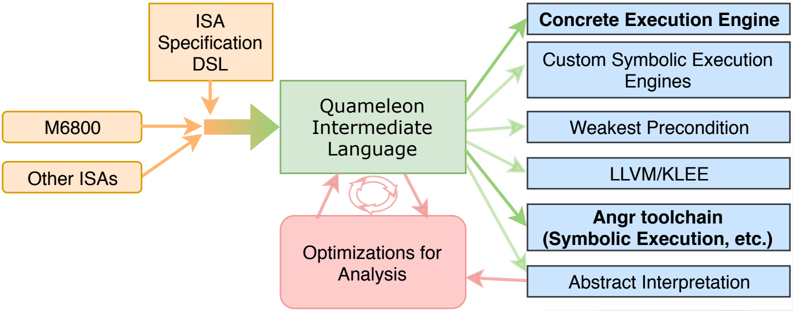
Motivation

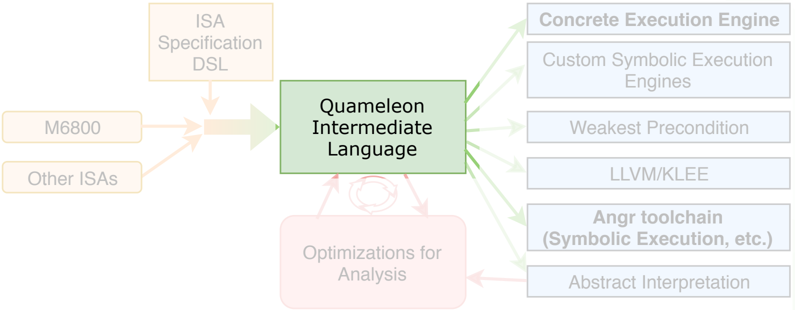
- Need to analyze binaries on old, obscure ISAs
  - ISAs not supported by existing tools
  - No machine-readable specification
  - Bad old days: No IEEE 754 floats, no 8-bit bytes
- Other tools gain lots of efficiency from expressive ISAs and feature-rich Intermediate Languages (ILs)
- We instead require an adaptable IL

Fun example: cLEMENCy ISA invented for DEFCON had 9-bit bytes, 27-bit words, middle-endian [9]

Architectural Overview



Architectural Overview



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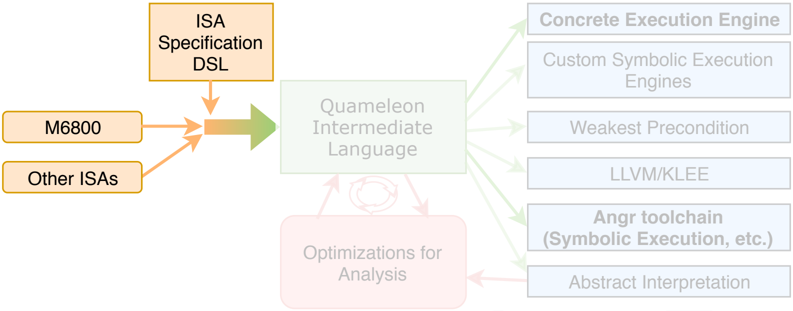
Design Goals of the Quameleon Intermediate Language (QIL)

- Sound analysis of binaries
- Lift binaries into a simple IL amenable to multiple analysis backends
- Close to LLVM IR in spirit
- Size of QIL ($\sim$ 60 instructions) means easy to manipulate, harder to write
- Balance this with Haskell as a macro-assembler for QIL

Quameleon Intermediate Language (QIL)

- Static Single Assignment (SSA)
- Program consists of a list of blocks, single entry, multiple-exit
- Data are stored in bit vectors of parametrizable width
- Can read/write to locations like RAM, registers
- Keep track of I/O as sequence of reads/writes

Haskell Embedded Domain Specific Language (DSL)



Sample M6800

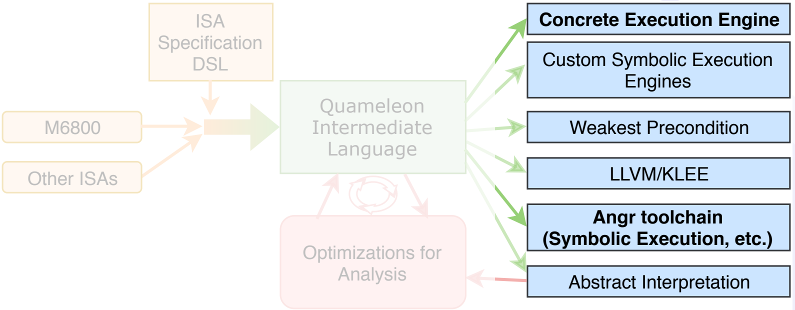
```
A <— 0xE
A <— A & [0x40]
```

We want to match the manual precisely

. . . and Its Corresponding Semantics

```
AND r l -> do
  ra <- getRegVal r
  op <- loc8ToVal l -- Loc. of 8 bits in RAM
  rv <- andBit ra op
  z <- isZero rv
  writeReg r rv
  writeCC Zero z -- CC = Condition Code
  branch next
```

Back-ends


ISA Specification DSL
M6800
Other ISAs
Quameleon Intermediate Language
Optimizations for Analysis
Concrete Execution Engine
Custom Symbolic Execution Engines
Weakest Precondition
LLVM/KLEE
Angr toolchain (Symbolic Execution, etc.)
Abstract Interpretation

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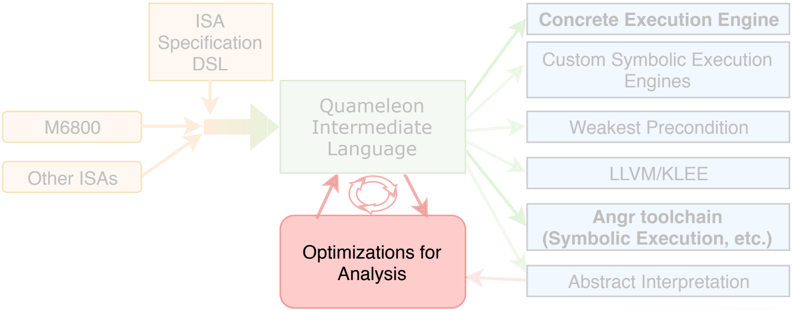
Current Back-ends

1. Emulator
2. Bridge to angr
   - angr is a symbolic execution engine primarily for cybersecurity
   - Treat QIL as an ISA that angr can execute

Optimizations



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QIL-QIL Optimizations

The goal is to facilitate analysis

- Constant folding
- Branch to known value
- Dead code elimination

}**Reduce code size**

- Inlining with simple heuristics
  e.g., inline everywhere
- Defunctionalization

}**Simplify CFG**

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1. How can we write specifications of instruction set architectures (ISAs) that enable static analysis for program verification?
2. How can we formalize and quantify the error from floating-point arithmetic in high-performance numerical programs?

A Statistical Analysis of Error in MPI Reduction Operations

Based off previously published work with Boyana Norris [7].

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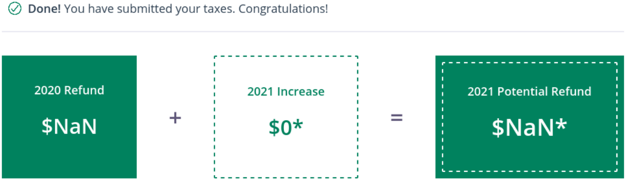
A Brief Introduction to Floating-Point Arithmetic

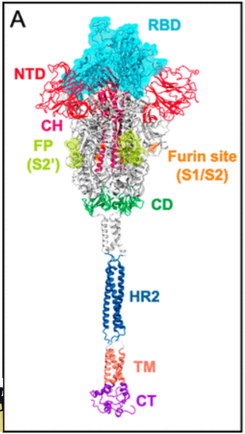
The rest of this talk focuses on floating-point (FP) arithmetic and floating-point operations (FLOPs)

← Ariane V, the $500 million overflow



We Don't Trust Floating Point

- Doesn't map perfectly to real numbers
- Can't even represent 1/10 exactly
- Complex behavior of error and exceptions



[1]

We Don't Trust Floating Point

- Doesn't map perfectly to real numbers
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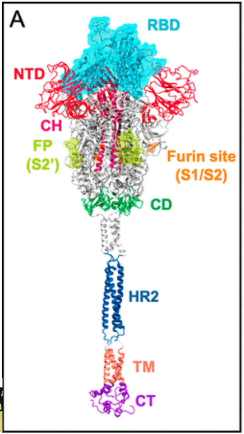
- Complex behavior of error and exceptions

> But it's what we're stuck with



[1]

Floating-Point Arithmetic Is Not Associative

- Let $\oplus$ be floating-point addition
- $0.1 \oplus (0.2 \oplus 0.3) =$ `0x1.3333333333334p-1`
- $(0.1 \oplus 0.2) \oplus 0.3 =$ `0x1.3333333333333p-1`
- Worse error when the magnitudes are different

Floating-Point Arithmetic Is Not Associative

Does this bit matter?

- Let $\oplus$ be floating-point addition
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Absolute vs. Relative Error

Let $\hat{x}$ be an approximation for x. Then relative error is

$$\left|\frac{\hat{x} - x}{x}\right|$$

and absolute error is

$$|\hat{x} - x|$$

- Think of absolute error as financial calculations; off by at most 1/10 cent (one mill)
- Think of relative error as significant digits

Bound on Relative Error

- Let $\cdot$ be one of $\{+, -, \div, \times\}$ and $\odot$ be its corresponding floating-point operation. Then

$$x \cdot y = (x \odot y)(1 + e) \text{ where } |e| \leq \epsilon. \qquad (1)$$

- For double-precision $\epsilon = 2^{-53}$
- This holds only for $x \odot y \neq 0$ and normal (not subnormal)

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Message Passing Interface (MPI)

- An API for communication between computers
- *de facto* standard for high-performance computing (HPC)
- Both "too high-level and too low-level" [8]

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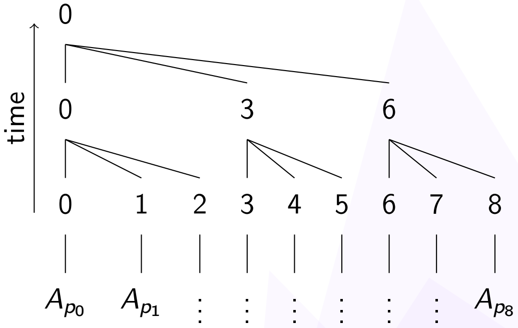
MPI Reduce

- Assume an array A of size n
- Reduce A to a single value
  - e.g. `MPI_SUM`
- Distribute A across MPI ranks (each p_k)
- Unspecified but usually deterministic reduction order on the same topology



How many ways are there to do this reduce?

- Depends on how we define acceptable reduction strategy
- We list four families
  1. Canonical Left-Associative (Canon)
  2. Fixed Order, Random Association (FORA)
  3. Random Order, Random Association (RORA)
  4. Random Order, Left-Associative (ROLA)

1. Canonical Left-Associative

- Left-associative
- Unambiguous: one reduction strategy
- No freedom to exploit parallelism

```
double acc = 0.0;
for (i = 0; i < N; i++) {
    acc += A[i];
}
```

The MPI Standard is Flexible

- Operations are assumed to be associative and commutative.
- You may specify a custom operation where commutativity is fixed (but not associativity)

Reduction Families Permitted by MPI

2. Fixed Order, Random Association (FORA)
3. Random Order, Random Association (RORA)
   - Default if you call `MPI_Reduce`
4. Random Order, Left Associative (ROLA)
   - To compare with previous work [3]

- All of these have at least an exponential number of *associations*
- We generate these by shuffling an array, then generating random trees with Rémy's Procedure [4, § 7.2]

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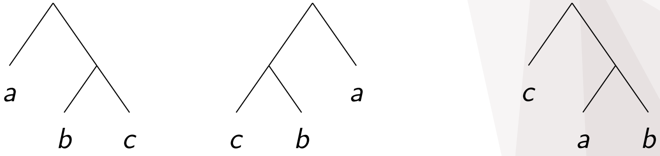
Example Summation

With the commutative but nonassociative operator $\oplus$,
$r_1 = r_2$ but $r_2 \neq r_3$.

$r_1 = a \oplus (b \oplus c)$ $r_2 = (c \oplus b) \oplus a$ $r_3 = c \oplus (b \oplus a)$



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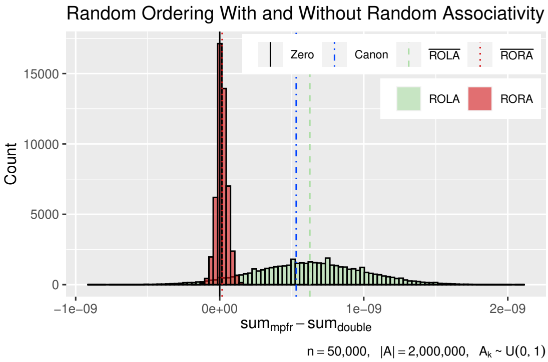
Absolute Error

Let $\sum^{\oplus}$ be floating point sum, S_A be the true sum.
Wilkinson back in '63 proved summation error is bounded by

$$\left| \sum_{k=1}^{\oplus\ n} A_k - S_A \right| \leq \epsilon(n-1) \sum_{k=1}^{n} |A_k| + O(\epsilon^2). \tag{2}$$

Left and Random Associativity (ROLA vs. RORA)

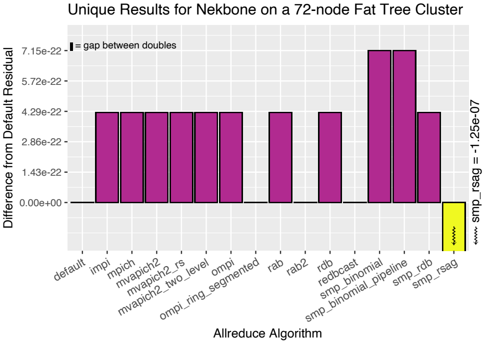
- Histogram of error
- 1000-digit float (MPFR) is true value
- ROLA is a biased sum
- worst RORA has smaller error than canonical



Bound from (2): 4.44×10^{-4}

Nekbone

- Nekbone is a computational fluid dynamics proxy app
- We look at residual of conjugate gradient
- We use SimGrid [2] to try out 16 different allreduce algorithms



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The Challenges of Floating-Point

Suppose we want a safe floating-point divide? Easy, right?

```
float unsafe(float x) {
    if (x==0.0)
        return 0.0;
    else
        return 1.0 / x;
}
```

The Challenges of Floating-Point

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        return 1.0 / x;
}
```

wrong

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Truly Safe FP Divide

```
#include <math.h>
float reallysafe(float x) {
    // Cast to int without changing bits
    unsigned long c = *(unsigned long*)&x;
    if (isnan(x) || isinf(x)
            || (0x80000000 <= c && c <= 0x80200000)
            || (0x00000000 <= c && c <= 0x00200000))
        return 0.0;
else
    return 1.0 / x;
}
```

Existing Static Error Analysis

- Tools like FPTaylor, Satire, Daisy
- Take as input a DSL describing a FP program and rages of its inputs
- Output maximum possible error, found with global optimziation
- No loops or conditionals
- Slow: $\sim$ 1.5 hours for 500 FLOPs
- Most are sound

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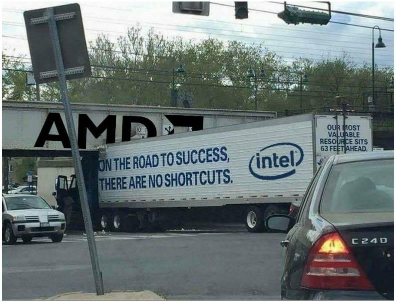
Why We Should Care About Soundness



- Underapproximating error may be worse than overapproximating

Subnormal Numbers

We previously saw ϵ, the bound on relative error. For very small numbers, we must also define an absolute error

$$(x \odot y) = (x \cdot y)(1 + e) + d$$

where $|e| \leq \epsilon$, $|d| \leq \delta$.
e.g., $\delta = 2^{-1074}$ for double-precision

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Motivation: Vector Normalization

Given a vector x, compute

$$q = \frac{x}{\|x\|_2}$$

Do this by multiplying each x_i by $1/\sqrt{|x \cdot x|}$.

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Dot Products

Define

$$\gamma_n = \frac{n\epsilon}{1 - n\epsilon}.$$

Unsound (existing bound)

$$|\langle x, y\rangle - \mathrm{flt}(x \cdot y)| \leq \gamma_n |x| \cdot |y| \tag{3}$$

Our improvement

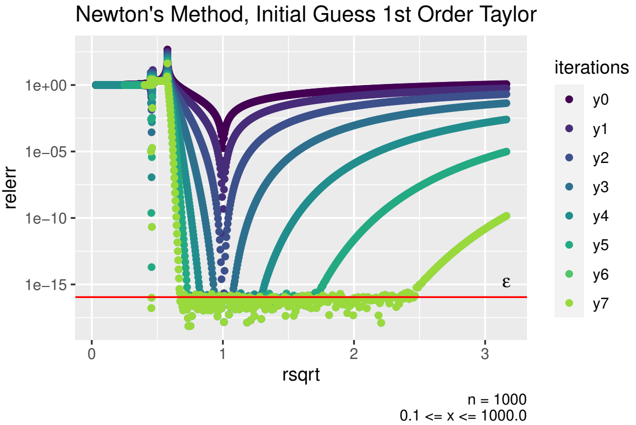
$$|\langle x, y\rangle - \mathrm{flt}(x \cdot y)| \leq \gamma_n |x| \cdot |y| + n\delta(1 + \gamma_{n-1}), \tag{4}$$

My Key Insight

> Combine global search for the hard parts and computed bounds for the majority of FLOPs

- FPTaylor on 500 FLOPs: 55,000 seconds
- FPTaylor + (4) on 10^9 FLOPs: 10 seconds
- Speedup of 10^{11} - Not bad!
- Need to compare with empirical error

Reciprocal Square Root



Input range and quality of initial guess have a large effect on convergence

Well. . . Did I Answer It?

How can we apply high-level reasoning techniques about computer programs to low-level implementations? Specifically,

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Future Research Directions

- Binary Analysis
- The Emerging Field of Formal Numerical Methods
  - Blend probabilistic and deterministic error analysis
- Precomputation, Once Again

Conclusion

- Verification of low-level programs is hard
- My techniques rely on detailed mathematical models and the speed of modern computers
- They help people write correct, fast code
  - Quameleon: enables binary analysis on uncommon ISAs
  - A statistical analysis of error for parallel reduction algorithms
  - A sound analysis of error for optimized math kernels to quantify the performance-accuracy tradeoff

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https://sampollard.github.io/research

Thank you!

Motivation for Precomputation: *Quake III: Arena*

```
float Q_rsqrt(float number) {
    long i;
    float x2, y;
    const float threehalfs = 1.5F;
    x2 = number * 0.5F;
    y = number;
    i = *(long *) &y;
    i = 0x5f3759df - (i >> 1);
    y = *(float *) &i;
    y = y * (threehalfs - (x2*y*y));
    return y;
}
```

- "Magic" constant `0x5f3759df` precomputed for efficiency [5]

Motivation for Precomputation: *Quake III: Arena*

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```

- "Magic" constant `0x5f3759df` precomputed for efficiency [5]

What does this do to a real number?

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